Also solved by C. A. BARNHART, H. HALPERIN, H. L. OLSON, and A. PELLETIER.

2773 [1919, 212]. Proposed by JOSEPH ROSENBAUM, Milford, Conn.

Point out the fallacy in the proof of the following problem:

In the triangle $A_1B_1C_1$ let m be a point such that the sum of the distances from it to the sides is a maximum; also let $A_2B_2C_2$ be a triangle formed by drawing lines through the vertices A_1, B_1, and C_1 parallel to their opposite sides. Then the sum of the distances from m to the sides of the triangle $A_2B_2C_2$ is a minimum.

Proof.—Because the sides of the two triangles are parallel in pairs, the sum of the distances from a variable point P in triangle $A_1B_1C_1$ to the six sides of the two triangles is constant. Now by hypothesis M is a point for which one part of this constant sum is a maximum, and hence it follows that the other part is a minimum.

SOLUTION BY H. L. OLSON, University of Wisconsin.

This proof is correct, with the understanding that if a point P is on the opposite side of BC, for example, to the vertex A, the distance to the side BC is to be regarded as negative. It is easy to see, however, that the point M does not exist, and that the proposition is therefore vacuous. Represent the perpendicular distances from P to the sides BC, AC, and AB by α, β, and γ respectively. If we denote by Δ the area of the triangle ABC, we are to minimize the function $\alpha + \beta + \gamma$, subject to the condition $a\alpha + b\beta + c\gamma = 2\Delta$. ($a$, b, and c represent, as is customary, the sides BC, AC, and AB, respectively.) Eliminating γ, we have, as the function to be minimized,

$$\left(1-\frac{a}{c}\right)\alpha+\left(1-\frac{b}{c}\right)\beta+\frac{2\Delta}{c}.$$

Hence, the derivatives, $\left(1-\frac{a}{c}\right)$, and $\left(1-\frac{b}{c}\right)$, of this function with respect to α and β must vanish; but for the general triangle they do not vanish and hence M does not exist. If, however, $a = b = c$, the sum of the distances is the constant $2\Delta/c$; likewise the sum of the distances for the corresponding triangle $A_2B_2C_2$ is constant.

Also solved by A. PELLETIER and A. L. WECHSLER.

2774 [1919, 212]. Proposed by FRANK IRWIN, University of California.

Evaluate the circulants

$$\begin{vmatrix} 1 & 2 & 3 & \cdots & n-1 & n \\ n & 1 & 2 & \cdots & n-2 & n-1 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ 2 & 3 & 4 & \cdots & n & 1 \end{vmatrix}, \quad \text{and} \quad \begin{vmatrix} a_1 & a_2 & a_3 & \cdots & a_{n-1} & a_n \\ a_n & a_1 & a_2 & \cdots & a_{n-2} & a_{n-1} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ a_2 & a_3 & a_4 & \cdots & a_n & a_1 \end{vmatrix},$$

where, in the latter, $a_1, a_2, \cdots, a_n$ form an arithmetical progression.

I. SOLUTION BY P. J. DA CUNHA, University of Lisbon, Portugal.

Denote the first of these circulants by Δ and the second by Δ^4. Let

$$s_n = \frac{1+n}{2}n$$

be the sum of the first n positive integers. Add to the elements of the last line of Δ the sum of the corresponding elements of all the preceding lines. We obtain a determinant which we can write as the product

$$\Delta = s_n \begin{vmatrix} 1 & 2 & 3 & \cdots & n-2 & n-1 & n \\ n & 1 & 2 & \cdots & n-3 & n-2 & n-1 \\ n-1 & n & 1 & \cdots & n-4 & n-3 & n-2 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ 4 & 5 & 6 & \cdots & 1 & 2 & 3 \\ 3 & 4 & 5 & \cdots & n & 1 & 2 \\ 1 & 1 & 1 & \cdots & 1 & 1 & 1 \end{vmatrix}$$